Principal component analysis on images

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DTU Compute

Based on

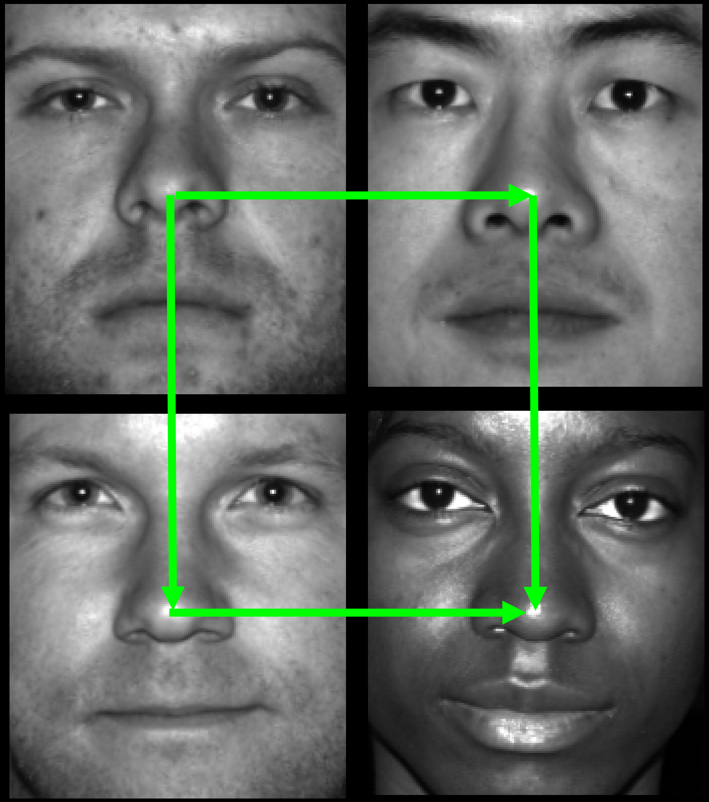
M. Turk and A. Pentland. *Face recognition using eigenfaces*. Computer Vision and Pattern Recognition, 1991.

http://compute.dtu.dk/courses/02502

Principal Component Analysis on images learning objectives

- Construct a column matrix from a single gray scale image
- Construct a data matrix from a set of gray scale images
- Compute and visualize an average image from a set of images
- Compute the principal components of a set of images
- Visualize the principal components computed from a set of images
- Synthesize an image by combining the average image and a linear combination of principal components

Face data



- 38 face images
  - 168 x 192 grayscale
- Aligned
  - The anatomy is placed ”in the same position in all image”
- Same illumination conditions on the images we use

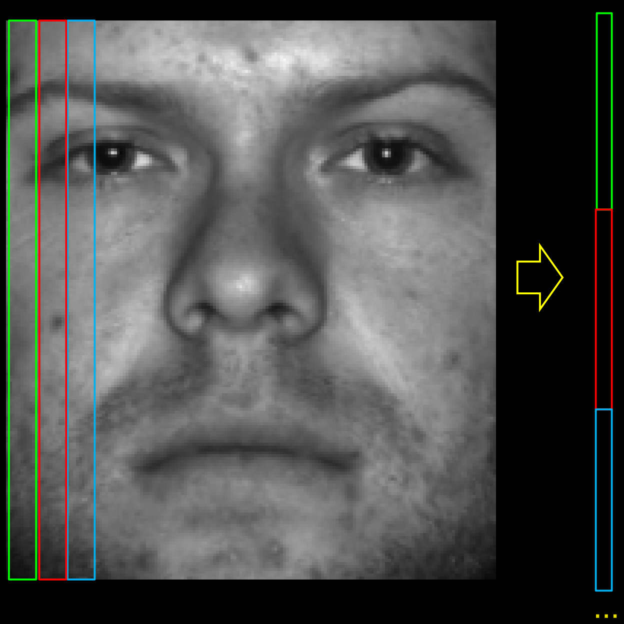
The Extended Yale Face Database B
http://vision.ucsd.edu/~leekc/ExtYaleDatabase/ExtYaleB.html

Principal component analysis on face images

- What is the main variation in face images?
  - The variation of appearance
  - Not the position in the image
  - Not the light conditions
  - Not the direction of the head

Putting images into matrices

- An image can be made into a column matrix
  - Stack all image columns into one column



$$I = \begin{bmatrix} p_1 \\ p_2 \\ \dots \\ p_m \end{bmatrix}$$

Face images in matrix form

- One column is one face
- n=38 faces
- m=168x192 = 32256 pixel values per image

$$X=\begin{bmatrix} p_{1,1} & \cdots & p_{1,n} \\ \vdots & \ddots & \vdots \\ p_{m,1} & \cdots & p_{m,n} \end{bmatrix}$$

The average face

$$X = \begin{bmatrix} p_{1,1} & \cdots & p_{1,n} \\ \vdots & \ddots & \vdots \\ p_{m,1} & \cdots & p_{m,n} \end{bmatrix}$$

- The average face
  - Average of each row
  - One column
  - Put it back into image shape
- Blurry around the eyes
  - Not perfectly aligned

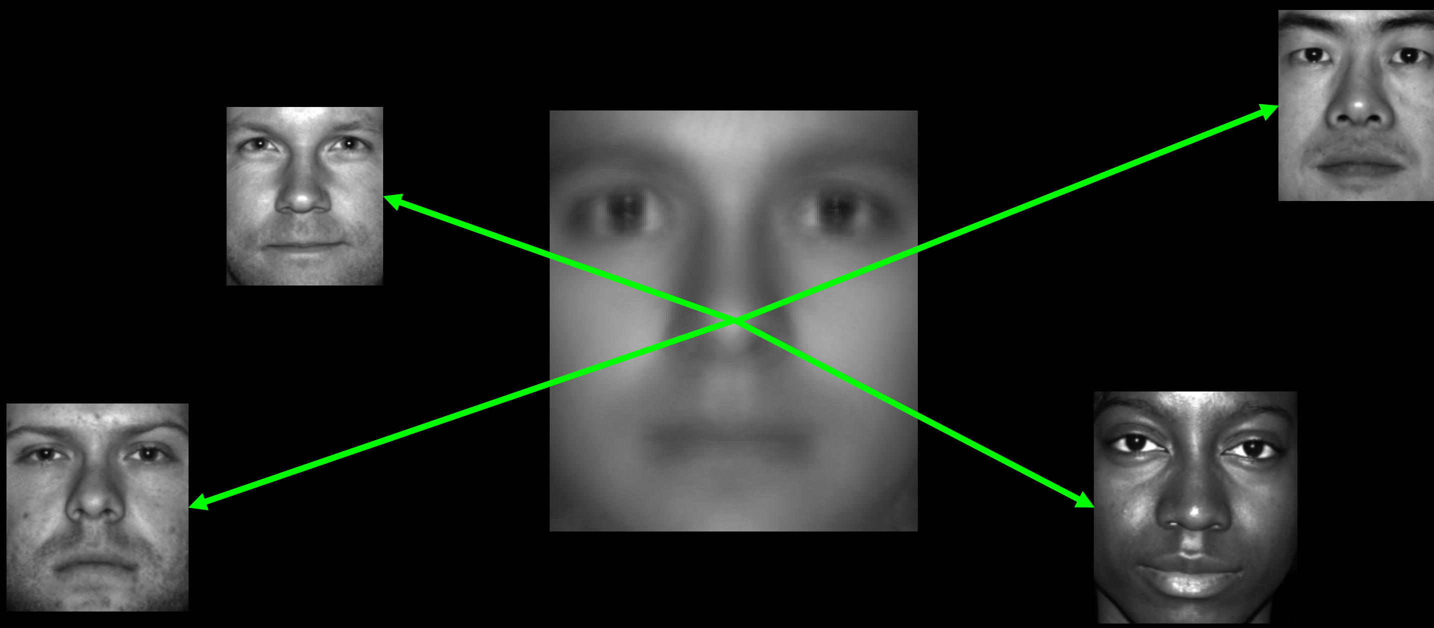
Subtracting the mean face

$$X' = \begin{bmatrix} p_{1,1} & \cdots & p_{1,n} \\ \vdots & \ddots & \vdots \\ p_{m,1} & \cdots & p_{m,n} \end{bmatrix} - \bar{X}$$

- We subtract the mean face from all faces

Analyzing the deviation from the mean face

- We want to do the principal component analysis on the *deviations from the average face*

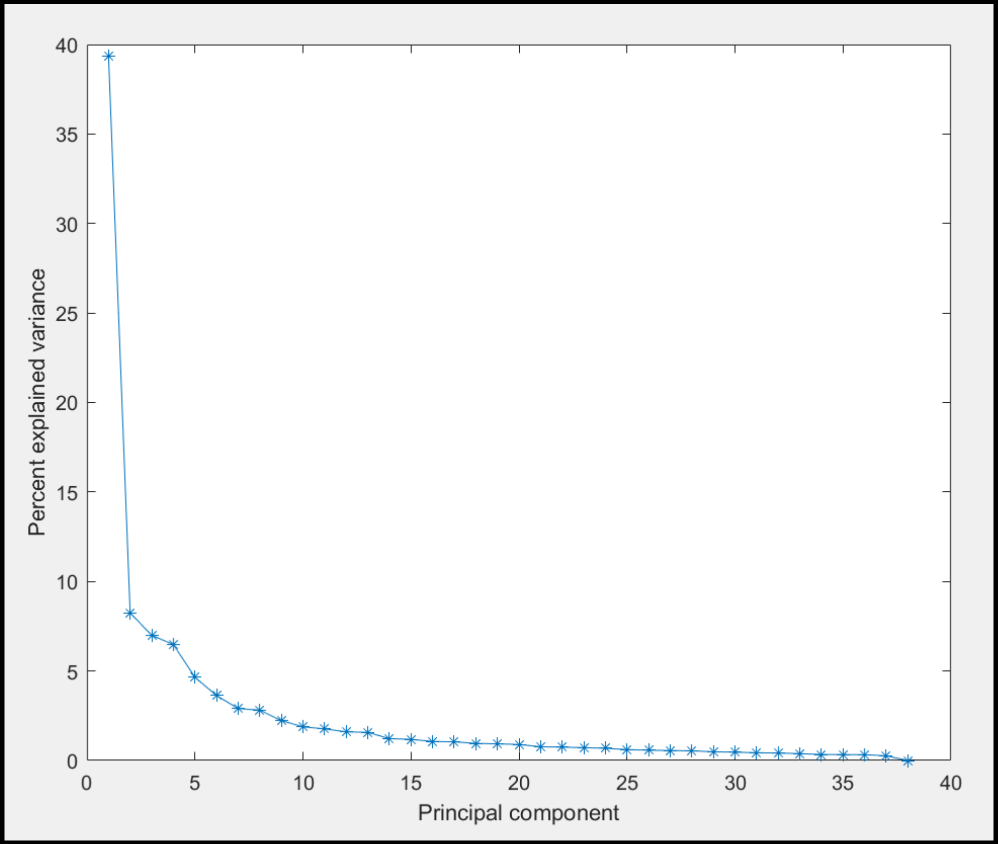


PCA Analysis on face data

$$X' = \begin{bmatrix} p_{1,1} & \cdots & p_{1,n} \\ \vdots & \ddots & \vdots \\ p_{m,1} & \cdots & p_{m,n} \end{bmatrix} - \bar{X}$$

- We do the PCA analysis on the X’ matrix
- X’ is 32256 x 38
- Standard covariance matrix is 32256 x 32256
- Turk and Pentland found a trick:
  - Compute the PCA on the 38 x 38 matrix instead of the 32256x32256 matrix
  - Details in the paper
    - Beyond the scope here

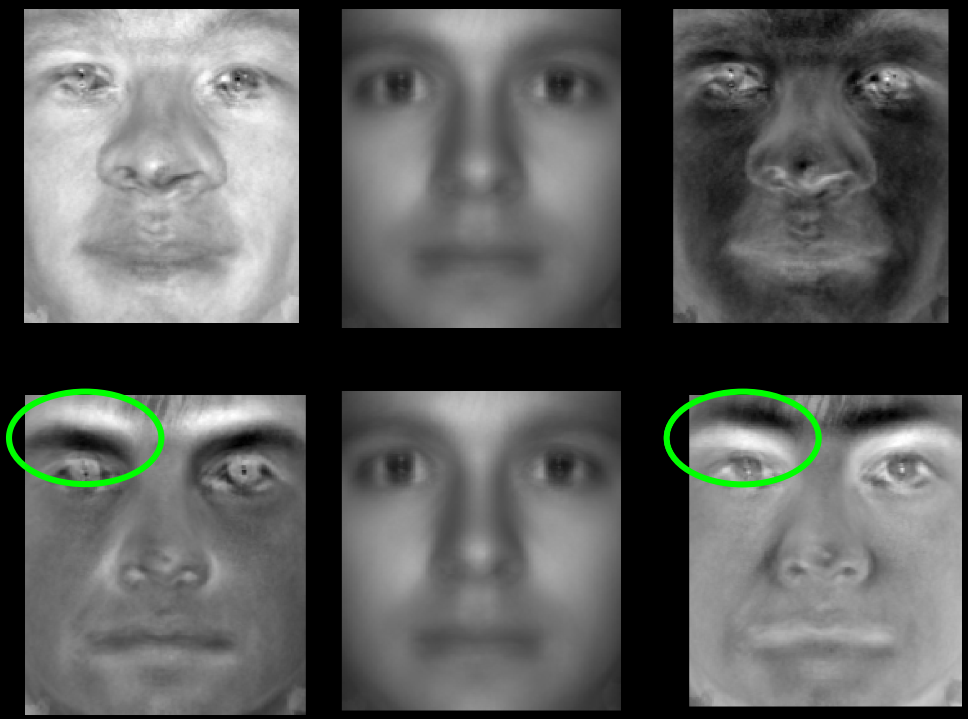
PCA on faces



- First eigenvector explains 40% of variation
- Second eigenvector explains 8% of variation

Visualizing the PCA faces

Main deviations from the average face



-PC

Average face

+PC

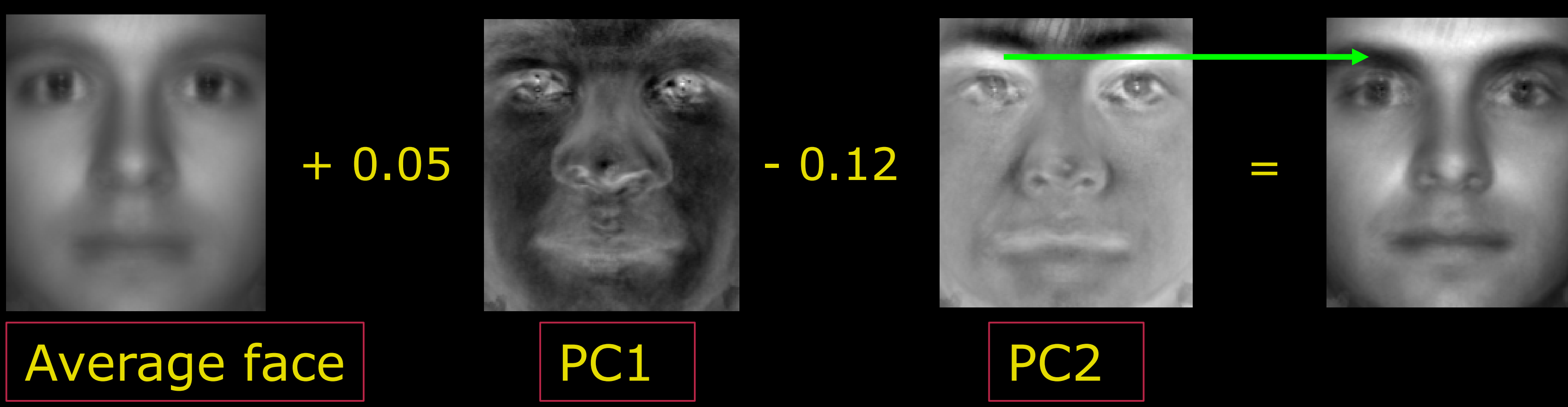
First PC – 40% of variation

Second PC – 8% of variation

A tool to see major variations – brow lifting

Synthesizing faces

- A new face can be created by combining
  - Average face
  - Linear combination of principal components



Decomposing faces

- A given face can be reconstructed using
  - The average face
  - Linear combination of principal components
- Found by projecting the face on the principal components
- The weights can then be used for classification/identification

$+w_1$

$+w_2$

$\cong$

Average face

PC1

PC2

Face analysis plus plus?

- More examples later in the course